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IDENTIFICATION OF THE PARAMETERS OF A NESTED CYLINDRICAL HEAT SOURCE UNDER STATIONARY SELF-HEATING OF A RAW MATERIAL MASS OF THE SAME FORM

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This paper describes a computer method for determining the coordinate of the center, the heat-release power, and the sizes of a homogeneous internal cylindrical heat source in a stationary axially symmetric temperature field arising under self-heating of a plant raw material.

Inverse heat-conduction problems (IHP) have been the subject of numerous investigations. The developed methods of their solution and the results obtained have appeared in monographs [1–9] and other publications. The aim of these investigations was mainly to solve the heat-conduction problems that arise in the aircraft and rocket industry, the power-engineering industry, in foundry, and in thermal treatment of materials.

In the last few years, it has become clear that increasing fire safety is also associated with the solution of IHPs. The location of self-heating sites of raw materials and the determination of their parameters by the results of temperature measurements at individual points of the mass permit using means of address supply of cooling and fire-fighting substances, which considerably simplifies the elimination of emergency situations. Along these lines, investigations where the parameters of internal heat sources are determined by the method of successive narrowing of given intervals have been made [10–12]. Below, using the above method, identification of four parameters in reconstructing the temperature field in a cylindrical raw material mass is carried out. We solve the inverse problem in the following formulation.

Assume that a cylindrical raw material mass of radius R_c and height l has a zero excess temperature $T = T(r, z)$ on the end $z = 0$ and on the lateral surface $r = R_c$ over the ambient temperature, which we assume to be a constant. The lower end of the mass $z = 1$ is perfectly thermally insulated. The heat conductivity coefficient of the raw material is constant and equal to λ. On the cylinder axis the center of a homogeneous heat source of the same form is situated. It has a radius R and a height (thickness) $2H$. The upper end of the heat source is separated from the upper end of the mass by a distance $\zeta = H$ (see Fig. 1). The specific heat-release power in the heat source is constant and equal to q_0. Outside it it is equal to zero.

According to the assumptions made, the excess temperature distribution in the raw material obeys the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = -\frac{q_0}{\lambda}\left[\omega(z - \zeta + H) - \omega(z - \zeta - H)\right]\omega(R - r)$$

and the boundary conditions

$$T(r, 0) = T(R_c, z) = 0\,, \quad T_z'(r, l) = 0\,.$$

In these, $\omega(t)$ is a Heaviside function; the prime means a derivative with respect to z.

Four parameters, q_0, R, H, and ζ, are the unknown quantities and are to be identified. The initial information for their identification is the excess temperatures

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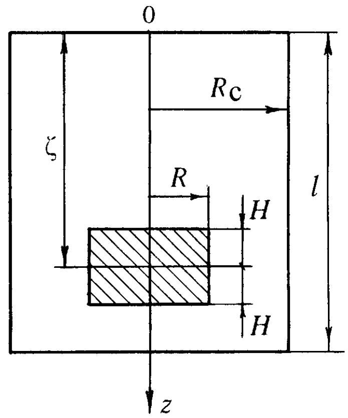

Fig. 1. Design diagram.

$$T_{im} = T(0, z_i)\,, \quad z_i = z_0 + i\Delta z\,, \quad i = \overline{0; (K-1)}\,,$$

measured at K points on the central vertical axis of the mass. Note that to make an identification, the temperatures can also be measured on any other vertical axes in the raw material, but this will somewhat complicate the calculations.

To determine the above parameters, we make use of the solution of the direct problem of stationary heat conduction in the form of a single Fourier–Bessel series [13] that satisfies the given heat-exchange conditions:

$$T(r, z) = \frac{2q_0 R R_c}{\lambda} F(r, z, R, H, \zeta)\,. \tag{1}$$

Here

$$F(r, z, R, H, \zeta) = \sum_{m=1}^{\infty} \frac{J_1(\gamma_m R)}{s_m^3 J_1^2(s_m)} f_m(z)\, J_0(\gamma_m r)\,; \tag{2}$$

$\gamma_m = s_m R_c^{-1}$; s_m $(m = \overline{1; \infty})$ is a sequence of positive roots of the equation $J_0(s) = 0$;

$$f_m(z) = \frac{2}{\cosh(\gamma_m l)} \cosh(\gamma_m (l - \zeta)) \sinh(\gamma_m H) \sinh(\gamma_m z) \quad \text{for} \quad 0 \le z \le \zeta - H\,;$$

$$f_m(z) = 1 - \frac{1}{\cosh(\gamma_m l)} [\cosh(\gamma_m (l - z)) \cosh(\gamma_m (\zeta - H)) + \sinh(\gamma_m z) \sinh(\gamma_m (1 - \zeta - H))] \quad \text{for} \quad \zeta - H \le z \le \zeta + H\,;$$

$$f_m(z) = \frac{2}{\cosh(\gamma_m l)} \cosh(\gamma_m (l - z)) \sinh(\gamma_m H) \sinh(\gamma_m \zeta) \quad \text{for} \quad \zeta + H \le z \le l\,.$$

Expressions (1) and (2) are taken as an approximation dependence with varied parameters: R, H, and ζ. We determine the latter so that the sum of the squared differences of measured temperatures T_{im} and calculated temperatures $T_{ical} = T(0, z_i)$ is minimum. To this end, we make use of the algorithm proposed in [12]. It includes the following computer steps:

1) transition to the dimensionless values of temperatures $\overline{T}_{im} = T_{im} T_{0m}^{-1}$ is made;

2) the interval $R \in (R_{in}, R_{end})$ is given and divided into J parts, and in the external cycle by J from 0 to J $R_j = R_{in} + j(R_{end} - R_{in})J^{-1}$ are calculated;

3) the interval $H \in (H_{in}, H_{end})$ is given and divided into N parts, and in the first interval cycle by n from 0 to N $N_n = H_{in} + n(H_{end} - H_{in})N^{-1}$ are calculated;

TABLE 1. Values of the Parameters of the Center of Self-Heating Calculated at Different M

M	R, m	H, m	ζ, m	q_0, W·m^{-3}
10	0.219	1.614	4.279	48.802
20	0.241	1.613	4.280	40.471
40	0.237	1.613	4.280	41.762
160	0.235	1.613	4.280	42.352

TABLE 2. Values of R, H, ζ, and q_0 Calculated at Different M

M	R, m	H, m	ζ, m	q_0, W·m^{-3}
3	1.618	0.724	10.382	3.985
10	1.608	0.722	10.382	4.034
20	1.600	0.722	10.382	4.063
40	1.601	0.722	10.382	4.061

TABLE 3. Excess Temperatures near the Center of Self-Heating

z, m	10.38	10.45	10.50	10.55	10.60	10.70
T, °C	48.90	49.05	49.06	48.97	48.79	48.17

4) the interval $\zeta \in (\zeta_{in}, \zeta_{end})$ is taken and divided into P parts, and in the second internal cycle by p from 0 to P the following quantities are calculated: a) $\zeta_p = \zeta_{in} + p(\zeta_{end} - \zeta_{in})P^{-1}$; b) $F_i = F(0, z_i, R_j, H_n, \zeta_p)$; c) $S = \sum_{i=0}^{k-1} (\overline{T}_{im} - F_i F_0^{-1})^2$;

5) in the course of the calculations, those values of R_j, H_n, and ζ_p are stored to which there corresponds the least sum S; they are taken to be the first approximation in the problem solution;

6) to redetermine the parameters, in the vicinity of the found values of R_j, H_n, and ζ_p shorter intervals of solution insulation are given and the calculation is repeated;

7) upon reaching the needed accuracy of parameter determination, $R \approx R_j$, $H \approx H_n$, and $\zeta \approx \zeta_p$ are taken, and by the formula

$$q_0 = \lambda T_{0m} \left(2RR_c F(0, z_0, R, H, \zeta)\right)^{-1} \tag{3}$$

the specific heat-release power in the heat source is calculated;

8) the results of the identification are substituted into expressions (1) and (2) and $T_{ical}(0, z_i)$ at measurement points are calculated.

This gives additional information on the uniformity of the T_{ical} approximation to T_{im}. The approximation on the average can be judged from the value of min S.

If, in giving the solution insulation intervals, an error (blunder) is made, then min S is attained at the edge of the interval. In such a case, the latter should be widened (or displaced) towards adequate values.

The choice of points for temperature measurements on the cylinder axis promotes calculations, since $J_0(0) = 1$ and in expansion (2) we do not have to calculate $J_0(\gamma_m r)$.

Let us analyze with two examples the results of the identification made for a mass of grain having [14] $\lambda = 0.06$ W/(m·K), $R_c = 3$ m, and $l = 12$ m.

Example 1. Calculate the parameters of the heat source at which on the mass axis at points with z coordinates $z_i = 3 + 0.6i$ m, $i = \overline{0; 5}$, temperatures T_{im}: 43, 50, 52, 51, 46, 22°C arise.

We make the identification, holding in the partial sum of series (2) 20 terms.

Assuming $R \in (0.1, 0.6)$ m, $H \in (0.8, 1.8)$ m, at $J = N = 20$, $P = 30$, by the above computer method we find: $R_j \approx 0.2$ m, $H_n \approx 1.6$ m, $\zeta_p \approx 4.3$ m, min $S \approx 5.5 \cdot 10^{-4}$. To redetermine the results, we assume $R \in (0.15, 0.25)$ m, $H \in (1.55, 1.65)$ m, $\zeta \in (4.25, 4.35)$ m, and $J = N = P = 20$. After the calculation we obtain: $R_j \approx 0.24$ m, $H_n \approx 1.62$ m, $\zeta_p \approx 4.28$ m, min $S \approx 1.79 \cdot 10^{-5}$. We perform the second redetermination at $R \in (0.23, 0.25)$ m, $H \in (1.61, 1.63)$ m, $\zeta \in (4.27, 4.29)$ m, and $J = N = P = 20$. It yields: $R_j \approx 0.241$ m, $H_n \approx 1.613$ m, $\zeta_p \approx 4.280$ m, and min $S \approx 1.434 \cdot 10^{-5}$. Restricting ourselves to the attained accuracy, by formula (3) we find $q_0 \approx 40.471$ W/m^3.

Substituting the identification results into the solution of the direct heat conduction problem, we obtain $T_{i\text{cal}}$: 43.00, 50.14, 51.93, 50.97, 46.02, and 21.98°C. The calculated temperatures are in good agreement with the initial values that were assumed for making the identification.

The results of the numerical solution of the IHP depend on the number M of held members in the partial sum of series (2) (see Table 1).

The investigation shows that the identification of H and ζ can be made at relatively small values of M. This makes it possible to promote (simplify) the calculations, i.e., take larger M for redetermining R and q_0.

Example 2. Let us see what the parameters of the heat source will be if on the mass axis at $z_i = 8 + i$ m and $i = \overline{1; 4}$ excess temperatures $T_{i\text{m}}$: 9, 21, 45, 44, and 32°C are attained.

In the partial sum of series (2) 10 terms are held.

Given $R \in (0.5, 2.9)$ m, $H \in (0.1, 1)$ m, $\zeta \in (6, 12)$ m, at $J = N = P = 30$ on computer, we obtain: $R_j \approx 1.8$ m, $H_n \approx 0.6$ m, $\zeta_p \approx 10.4$ m, and min $S \approx 5.9 \cdot 10^{-3}$.

To redetermine the results, we take $R \in (1.58, 1.98)$ m, $H \in (0.38, 0.78)$ m, $\zeta \in (10.2, 10.6)$ m, and $J = N = P = 40$. Having performed the calculation, we find: $R_j \approx 1.59$ m, $H_n \approx 0.73$ m, $\zeta_p \approx 10.38$ m, and min $S \approx 7.80 \cdot 10^{-4}$. We perform the second redetermination at $R \in (1.55, 1.63)$ m, $H \in (0.69, 0.77)$ m, $\zeta \in (10.34, 10.42)$ m, and $J = N = P = 40$. It yields: $R_j \approx 1.608$ m, $H_n \approx 0.722$ m, $\zeta_p \approx 10.382$ m, and min $S \approx 6.356 \cdot 10^{-4}$. Restricting ourselves to the above approximation, by formula (3) we find $q_0 \approx 4.034$ W/m^3.

According to the solution of (1) and (2), to the identification parameters there correspond the following calculated temperatures $T_{i\text{cal}}$: 9.00, 20.85, 44.94, 44.10, and 31.87°C. They are in fair agreement with those $T_{i\text{m}}$ that were given for solving the IHP.

The results of the identification with other numbers of terms of series (2) are presented in Table 2.

In the example under consideration, the heat-source radius R is much larger than in the previous one. An increase in R promotes the convergence of series (2). Therefore, a good accuracy of the identification in the second example takes place at smaller values of M compared to those required in Example 1.

Using the results of the temperature-field reconstruction, it is easy, using expressions (1) and (2), to find the maximum excess temperature in the raw material. On the spectral axis in the zone of the place of self-heating the values given in Table 3 have been obtained. From Table 3 it is seen that $T_{\max} \approx 49.06^{\circ}$C, and it is attained not at the center of the heat source but at a point displaced from the center towards the heat-insulated end of the mass.

NOTATION

r and z, radial and axial coordinates; $T(r, z)$, function of the temperature field; R_c and l, radius and height of the cylindrical raw-material mass; R and $2H$, radius and height of the place of self-heating; λ, heat-conductivity coefficient of the raw material; ζ, axial coordinate of the center of the place of self-heating; q_0, specific heat release in the place of self-heating; z_i, z-coordinates of points on the central axis; $T_{i\text{m}}$, measured temperature values; $T_{i\text{cal}}$, calculated temperature values; $J_0(t)$ and $J_1(t)$, Bessel functions of the first kind of indices zero and unity; s_m, mth positive zero of the function $J_0(s)$; S, sum of squared deviations. Subscripts: c, cylindrical; m, measured; cal, calculated; in, initial; end, end; max, maximum.

REFERENCES

1. O. M. Alifanov, E. A. Artyukhin, and S. V. Rumyantsev, *Extremum Methods for Solving Ill-Posed Problems and Their Application to Inverse Problems of Heat Transfer* [in Russian], Nauka, Moscow (1988).
2. O. M. Alifanov, *Inverse Problems of Heat Transfer* [in Russian], Mashinostroenie, Moscow (1988).

3. J. V. Beck, B. Blackwell, and Ch. Saint-Clar, Jr., *Ill-Posed Inverse Problems of Heat Conduction of Solids* [Russian translation], Mir, Moscow (1989).
4. L. A. Kozdoba and P. G. Krukovskii, *Methods for Solving of Inverse Problems of Heat Transfer* [in Russian], Naukova Dumka, Kiev (1982).
5. Yu. M. Matsevityi and A. V. Multanovskii, *Identification in Heat Conduction Problems* [in Russian], Naukova Dumka, Kiev (1982).
6. Yu. M. Matsevityi and S. F. Lushpenko, *Identification of Thermophysical Properties of Solids* [in Russian], Naukova Dumka, Kiev (1990).
7. N. I. Nikitenko, *Conjugate and Inverse Problems of Heat Transfer* [in Russian], Naukova Dumka, Kiev (1988).
8. A. G. Temkin, *Inverse Methods of Heat Conduction* [in Russian], Energiya, Moscow (1973).
9. N. V. Shumakov, *Method of Successive Intervals in Thermometry of Nonstationary Processes* [in Russian], Atomizdat, Moscow (1979).
10. V. P. Ol'shanskii, in: *Problems of Fire Safety* [in Russian], Collection of Sci. Papers of the Academy of Fire Safety of Ukraine, Issue 10, Folio, Khar'kov (2001), pp. 125–130.
11. V. P. Ol'shanskii, in: *Municipal Services of Cities* [in Russian], Collection of Sci.-Tech. Papers, Issue 38, Tekhnika, Kiev (2002), pp. 342–345.
12. V. P. Ol'shanskii, *Pozharovzryvobezopastnost'*, No. 3, 20–23 (2002).
13. V. P. Ol'shanskii and I. A. Krisa, in: *Integral Technologies and Energy Conservation* [in Russian], Quarterly published Sci.-Pract. Journal, No. 3, Nat. Tech. Univ. (KhPI), Khar'kov (2001), pp. 82–89.
14. L. P. Vogman, V. I. Gorshkov, and A. G. Degtyarev, *Fire Safety of Elevators* [in Russian], Stroiizdat, Moscow (1993).